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1 · R. W. Brigstocke 1880
Oct. 1893.

21

NOTES

—ON—

PRACTICAL ASTRONOMY

Compiled for the Use of the Cadets

I Class 1893 – 94

—OF THE—

ROYAL MILITARY COLLEGE OF CANADA

—BY—

Lieut.-Colonel J. R. OLIVER, R.A.,

Professor of Surveying and Military Topography.

KINGSTON:
PRINTED AT THE DAILY NEWS STEAM PRINTING HOUSE.
1880.

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CORRECTIONS AND ADDITIONS.

PAGE 5.—2d line above Fig. 2 should be "In the case of the altitude *of the sun* being measured.

PAGE 25.—2d line from top should be "=star's declination $\pm ZS$."

NOTE TO PAGE 7.—In the Nautical Almanac is given a method of finding the latitude by an altitude of the pole star when not on the meridian. The formula is:

$$\text{Latitude} = a - p \cos. h + \tfrac{1}{2} p^2 \sin.^2 h \tan. a.$$

When a is the star's altitude, h its hour angle, and p its north polar distance in circular measure.

NOTE TO PAGE 17.—Another way of finding the meridian very accurately is by taking transits of two stars of nearly the same Right Ascension, one of which should be as near the pole, and the other as far from it, as possible. The difference of their declination should not be less than 50°. The formula is:

$$d = \left\{ t' - t - (a' - a) \right\} \frac{\cos. \delta \cos. \delta'}{\cos. l \sin. (\delta - \delta')}$$

when d is the deviation (in horary units) l the latitude, t, a, and δ the sidereal times of transit, the R. A., and the declination, respectively, of the star near the pole, and t', a', δ' those of the other star. It is necessary to know the rate of the watch, but not its error, as the interval between the transits has simply to be corrected for rate and converted from mean into sidereal time.

If $t' - t - (a' - a)$ is positive the deviation will be east of north. If negative the deviation will be west.

NOTES ON PRACTICAL ASTRONOMY.

(The following notes are intended in great measure to serve the purpose of skeleton lectures, and presuppose a knowledge on the part of the student of the elementary facts of Astronomy and of the different methods of reckoning time. Blanks have been left for the necessary figures.)

In all extensive surveying operations it is absolutely necessary that the surveyor should know how to determine the variation of the compass; to lay down a meridian line correctly; to ascertain the true local mean time; and to find the latitude and longitude.

The instruments ordinarily used in the field for astronomical purposes are the sextant (with artificial horizon), the theodolite or transit, the portable transit telescope, and two or three good watches or chronometers. The pocket sextant is graduated to read single minutes and the large sextant reads to ten seconds of arc. The artificial horizon may be of mercury or molasses. In every case an observed altitude, whether obtained by the theodolite or sextant, should be corrected for index error before any further steps are taken. A set of mathematical tables and the Nautical Almanac for the year must always be at hand.

Latitude and Longitude are the co-ordinates by which the position of a point on the surface of a sphere is determined.

Taking the earth as a sphere, the length of a degree of longitude will be at the equator the same as that of a degree of latitude, but will diminish from the equator towards the poles, where it becomes nothing. The length of a degree of longitude at any place will be equal to that of a degree of latitude multiplied by the cosine of the latitude of the place.

For practical purposes it is as well to imagine the earth as a stationary sphere at the centre of the visible universe, and the heavenly bodies as projected on the surface of another sphere, having the same centre as the earth but at an infinite distance from it. The apparent motions of the sun, moon, and planets are measured and mapped on the surface of this great sphere, which latter apparently makes a complete revolution round the earth in a few minutes less than 24 hours. The fixed stars retain their relative places in the sphere, but the other heavenly bodies appear to move on its surface.

The co-ordinates by which the positions of the heavenly bodies on the sphere are determined are called Declination and Right Ascension: the first corresponding to Latitude, and the second to Longitude. There is, however, this difference—Longitude is reckoned eastwards 180° and westwards 180°, starting from a given meridian. Right Ascension is measured for the whole circle *eastwards* from 0″ to 360°, the zero point being one of the intersections of the equator and ecliptic, technically known as the "1st point of Aries."

THE LATITUDE.

The latitude of a place is the same as the altitude of the pole above the horizon at that place. It is north if the north pole is in sight, and *vice versâ*. It is usually found by the meridian altitude of a heavenly body, such as the sun or a star. If there is the slightest doubt it is safest to draw a figure, the plane of the paper representing the plane of the meridian. Let A be the place and S the heavenly body. We will suppose the altitude of the latter to be measured from the southern horizon. Draw HR tangential to the earth's surface at A to represent the horizon. Set off the angle RAS for the measured altitude, and from AS set off SAQ for the declination of the heavenly body as obtained from the Nautical Almanac. If the latter is south AQ will be above AS and *vice versâ*. The AQ will be the direction of the intersection of the meridian and equinoctial, and if we set off AP at right angles to AQ, P will be the position of the pole, and the angle HAP will be the latitude.

Fig. 1.

Now $HAP = 90° - RAS - SAQ = 90°$ — altitude — declination; or (if the declination is north) $= 90°$ + declination — altitude.

In the case of the altitude being measured above the horizon below the visible pole, as in figure 2, we shall have—

Fig. 2.

Lat. $= PAH$.
$= QAH - 90°$.
$= HAS + SAQ - 90°$.
$=$ Altitude $+$ declination $- 90°$.

This can only occur within the tropics.

The altitude may be obtained either by the theodolite or sextant. On land it is generally necessary to use an artificial horizon with the sextant. If the meridian altitude of the sun is so great that its double altitude is a larger angle than the sextant can measure a star must be observed instead, and this is rather a difficult thing to do. It is therefore, as a rule, better to employ the theodolite. It must, of course, be carefully levelled and the index error of the vertical arc allowed for. The pole star is a convenient star to employ on account of its slow motion, the radius of the circle it describes in the 24 hours being only $1°-20'$.

Fig. 3.

In figure 3 P is the pole, N the north point of the horizon HNR, S and S' the positions of the star when on the meridian at its upper and lower transits respectively. Then $SP=S'P=$ star's north polar distance or co-declination—SN or $S'N$ is the star's meridian altitude, and NP is the latitude. The time at which the star will be on the meridian must be found from the Nautical Almanac. The star's altitude is taken at the right time, and corrected for index error and refraction. Then we shall have, Latitude $=$ star's corrected altitude $\pm$ star's $N.P.D.$ according as the star is taken below or above the pole.

Fig. 4.

Should the star when at S be south of the zenith the case will be different. This is shown in figure 4 in which the plane of the paper represents the horizon. $WNES$ is the horizon—N and S its north and south points—Z the zenith—P the pole—S' and S'' the star. If we observe the star at S'' then its altitude will be $S''S$, and the latitude (or NP) will be $NS-PS''-S''S$

or $=180°-$ star's NPD $-$ altitude,

$=$ star's declination $+ 90° -$ altitude.

It must be remembered that all quantities which change from instant to instant, such as the equation of time and the sun's declination, and which have to be taken from the Nautical Almanac, must be corrected for longitude. For instance, suppose that the latitude of a place in the neighbourhood of Kingston had to be obtained by a meridian altitude of the sun. The declination of the latter is given in the N. A. for apparent noon at Greenwich, and when it is on the meridian of Kingston it is about 5h. 6m. P.M. apparent time at Greenwich, and the sun's declination for that hour must be obtained by a proportion.

TO FIND TEE TIME AT WHICH A STAR WILL BE ON THE MERIDIAN.

The data required for this are the star's Right Ascension, and the sidereal time of mean noon. The latter must be corrected as above for longitude and is the hour angle through which the 1st point of Aries has moved by 12 o'clock noon since it was last on the meridian.

In figure 5 let the plane of the paper represent the plane of the equator, P the pole, PM the meridian and γ the first point of Aries, and S the star. Then γS is the star's Right Ascension and γM the sidereal time of mean noon. Then the star will be on the meridian at an interval of sidereal time after mean noon corresponding to the arc SM, or $\gamma S - \gamma M$. If the star was at S', γM being greater than $\gamma S'$, it would have passed the meridian by an interval corresponding $S'M$, or $\gamma M - \gamma S'$. This interval must be reduced from sidereal to mean time and then deducted from 24.

Fig. 5.

Example.—To find at what time the pole star would be at its upper transit on a day when the sidereal time of mean noon, corrected for longitude, is 21h. 30m., the Right Ascension of the star being taken as 1h. 15m. Here the star has passed the meridian by an interval of 21h. 30m.—1h. 15m., or 20h. 15m., and will therefore be on the meridian in 3h. 45m. (sidereal) after noon.

Fig. 6.

The same result is also obtained thus:

From	24h.	0m.	
Subtract	21	30	=arc γSM
	2h.	30m.	=arc $M\gamma$
Add....	1	15	=arc γS
	3h.	45m.	=arc MS

As in the case of the latitude it is always safest to draw a figure.

The watch or chronometer used should be one whose rate of going can be depended on—that is, it ought to gain or lose the same amount in equal times.

In fixed observatories where the clock is regulated to show sidereal time, and the hours are numbered from 0 to 24, the clock's rate is

continually tested by star transits. But in the field it is not so easy either to check the rate or to keep the chronometer in good working order, since the mere moving about tends to derange its rate. If the watch gained or lost at a regular rate and its error at a certain instant was ascertained, then the true time could be found at any other instant by applying the rate. But no chronometer is perfect, and the best is always subject to two errors—the "Rate" and the "Error of Rate," the latter term meaning that the watch's rate is liable to vary from time to time. If the surveyor stays for a few days at any particular place he can always determine the rate at that time, and if he has more than one watch he can soon find out which has the least error of rate.

The chronometers used at sea are set to show Greenwich mean time. On land, if the longitude of a station and the true local mean time are known, Greenwich mean time can of course be at once found. One watch may be set to show Greenwich and the other local mean time.

GIVEN THE SIDEREAL TIME AT ANY INSTANT TO FIND THE MEAN TIME.

The Nautical Almanac gives the sidereal time of mean noon at Greenwich for every day in the year. That is, the sidereal time that has elapsed at 12 o'clock since the first point of Aries passed the meridian. To find the sidereal time of mean noon at any other place we must add or subtract 9s.8565 for each hour of longitude, according as the latter is west or east. Let T be the local sidereal time, t the sidereal time of the preceding mean noon, found as above. Then $T-t$ is the interval in sidereal time which has elapsed since mean noon. This, converted by tables into mean time, gives the hour.

For instance. In 75°, or 5h. west longitude on a certain day and hour we might have—

Sidereal time	21h.	9m.	24s.
Sidereal time of preceding noon at Greenwich..	18	47	0
	2	22	24
Subtract correction for longitude			49·28
Interval in sidereal time from mean noon......	2	21	34·72

which may be converted into mean time.

Conversely, when mean solar time is given and we want to find the sidereal time, we take the interval from preceding mean noon and convert it into sidereal time. Add to this the sidereal time of mean noon (taken from the N. A. and corrected for longitude) and we have the sidereal time.

Sidereal time is usually found by calculating the hour angle of a star from its observed altitude. This, added to the star's Right Ascension if the hour angle is west, or subtracted from it if east, gives the sidereal time.

TO FIND THE LOCAL MEAN TIME, AND THENCE THE LONGITUDE, BY AN OBSERVED ALTITUDE OF A HEAVENLY BODY.

To do this the latitude of the place must be known. The altitude should be taken at a time when the heavenly body is rapidly rising or falling—that is, when it is at some distance from the meridian. In figure 7 the circle is the horizon, P the pole, Z the zenith, S the heavenly body, SH its observed altitude. NPZ will be the meridian, ZPS will be a spherical triangle, in which PZ is the co-latitude of the place, PS the co-declination of the heavenly body, and ZS its co-altitude. The three sides of this triangle being thus known we can find the hour angle ZPS from the formula.

Fig. 7.

$$\text{Sin}^2 \tfrac{1}{2} ZPS = \frac{\text{Sin } (s-PS) \text{ Sin } (s-PZ)}{\text{Sin } PS \text{ Sin } PZ}$$

when s is the semi-perimeter of the triangle.

Dividing the hour angle by 15 will give its value in time—say T. If the body observed is the sun we shall now have the apparent solar time, and by adding or subtracting the equation of time we shall obtain mean time.

If the body is a fixed star or planet, from its known R. A., subtract T if the hour angle is east, or add it if it is west. This gives the sidereal time of the instant, from which the mean time can be inferred. The watch or chronometer time is noted at the instant the observation is taken. If the watch shows local mean time its error is at once obtained. If it shows Greenwich mean time the difference between this and the calculated local mean time gives the longitude.

20th February, 1880. Long. 5 h. 30m. west.

Hour angle of α Tauri was 2h. 30m. 17s. west, at 9 p.m. by watch. Find watch error :—

Star's R. A. =	4h.	29m.	4s.	
Hour Angle =	2	30	17	
	6	59	21	= Sidereal time of the instant.
Sidereal time of mean noon =	21h.	59m.	14s.	
Correction for longitude ..			55	
	22	0	9	
	24			
	1	59	51	
	6	59	21	
	8	59	12	= Sid'l interval since m.n.
	=8	57	44	in mean time.

Or the watch was 2m. 16s. fast.

(N.B.—A sidereal hour = a mean hour – 10 seconds, nearly.)

If the theodolite or transit is set up so that when the horizontal plate is clamped the telescope moves in the plane of the meridian the true time can be very easily found by noting the time of the transit across the central wire of the sun or a star. In the former case the mean of the transits of the east and west limbs is taken for that of the centre. This gives apparent noon, and by applying the equation of time mean noon can be found. As an example: The mean of the times of transit of the two limbs was 11h. 45m. by the watch on a day when the equation of time was 15m. 2s. to be subtracted from apparent time. To find the watch error—At the instant of transit it was apparent noon or 12h., and subtracting 15m. 2s. from this we have 11h. 44m. 58s. as the correct mean time of the instant. The watch was therefore 2 seconds fast.

If a star is used the case is different. The star's Right Ascension (found from the N.A.) is the sidereal time of its transit, and this has to be converted into mean time. For instance: On a certain day the transit of a star was observed at 8 p.m. The star's R.A. was 13h. 53m., and the mean time corresponding to sidereal time 13h. 53m. on that day was 8h. 3m. Hence the watch was 3 minutes slow.

THE MERIDIAN.

The meridian can be found with tolerable accuracy by the method of equal altitudes of a star. A still better way is by a theodolite observation of a circumpolar star (say Polaris) at its greatest elongation. Let P be the pole, S the star, Z the zenith. Then PZS is a spherical triangle, right-angled at S. PS is the star's co-declination, and PZ the co-latitude of the place. Sin PZS sin PZ=sin PS or, if l be the observer's latitude and d the star's declination,

$$\text{Sin } PZS=\frac{\cos d}{\cos l}$$

l need be only approximately known.

When the star is at its greatest elongation it will move for some time along the vertical wire of the theodolite. The time for this must be found from the Nautical Almanac and the equation (in the case of Polaris) is

Sidereal time of G.E.=Star's R.A.±6 hours.

Turn the axis of the telescope on the star, the horizontal plate being clamped at zero. Then make the vernier read the angle PZS, and, by means of a lantern, fix a picket exactly in the centre of the cross wires. This picket will be due north of the theodolite.

If the star's N. P. D. be considerable equation (*a*) will become—

Sidereal time of G. E.=star's R. A. $\pm < ZPS$ (in time), and the angle ZPS must be found from the equation

$$\text{Cos. } ZPS = \text{tan. N. P. D.} \times \text{tan. latitude.}$$

The star's altitude is given by the equation

$$\text{Sin. altitude} = \frac{\text{sin. latitude.}}{\text{cos. } N.P.D.}$$

The meridian may be still more accurately found by the superior and inferior transits of some star very near the pole, the clock rate having been found by transits of several stars on two successive nights. In the figure ZSS' is the vertical plane of the instrument, S and S' the positions of the star at the two transits. Let t and t' be the times of tne latter. We have to find the angle PZS. Treating the triangle PSS' as plane, we have: $PSS' + PS'S = APS' + SPZ$ $= 15\ (t' - t - 12\text{h})$ (in sidereal time).

Fig. 9.

Again, in triangle PZS we have

$$\text{Sin. } PZ \text{ sin. } PZS = \text{sin. } PS \text{ sin. } PSS'$$

$$\text{or cos. } l \text{ sin. } PZS = \text{cos. } d \text{ sin. } (15 \frac{t' - t - 12\text{h}}{2}$$

which gives PZS. If $t' - t$ be less than 12h. the line ZSS' will be west of ZP.

When the meridian has been accurately found a picket should be driven into the ground and the bearing of some well-defined object taken as a referring mark. For instance: at a picket in the Military College enclosure the spire of the Methodist Episcopal Church bears 76° 41′ west of true north.

THE DETERMINATION OF LONGITUDES BY SIGNALS.

Let A and B be two stations, and let the exact local time at each be known. Then if a phenomenon occurs of an instantaneous character which is visible at both places, the difference between the local times of its occurrence is the difference of the longitudes of the stations.

(*a*) From a point between A and B a rocket may be thrown up, and the instant of its explosion be noted at both places. By means of a series of such observations the difference between the longitudes of London and Paris was found.

(*b*) The eclipses of Jupiter's satellites also serve as signals. For this purpose the times of their occurrence are registered in the Nautical Almanac. They give very good approximations to the longitude, especially when both the disappearance and reappearance of the same satellite are visible.

(*c*) Instead of signals the stations may be connected by telegraph wires. Then it is possible, by proper mechanical contrivances, to register at *B* the time of a transit that takes place at *A*. The interval that elapses between that instant and the time of the transit at *B* will give a very correct value of the longitude; and if the observations are reciprocal any time lost in the transmission of the message will be corrected. If the correct local time is known at each station the comparison of the two at a given signal of course gives the difference of longitude. If two stations *A* and *B* are in direct telegraphic communication, and the longitude of *A* is known, it is evident that that of *B* can be determined with great precision providing the true local time at each is known.

TO FIND THE VARIATION OF THE COMPASS BY AN AMPLITUDE.

The amplitude of a heavenly body is at its angular distance from the east or west points of the horizon at its rising or setting.

Fig.10.

Let X be the heavenly body in the horizon. Its compass bearing is noted. Then in the quadrantal triangle PZX are given PZ=co-latitude, PX=co-declination or $N.P.D.$ of the object, and $ZX=90°$. We have to find the angle PZX, and thence its complement XZE the amplitude.

Cos. PX=sin. PZ cos. PZX.

or Sin. decl'n=cos. lat. sin. amplitude.

∴ Sin. amplitude=sin. decl'n. sec. lat.

Of course the difference between this calculated value of the amplitude and that given by the compass will be the variation.

TO FIND THE VARIATION BY AN AZIMUTH.

To do this the altitude of a heavenly body is observed with the sextant at the same instant that its compass bearing is noted.

Fig. 11.

Let X be the heavenly body at the instant of observation, P the pole, and Z the zenith. Then, in the triangle PZX ZX will be$=90°$ —altitude, PZ=the co-latitnde, and PX= the co-declination. The angle PZX will be the azimuth angle required, and is obtained from the equation—

$$\text{Sin.}^2 \tfrac{1}{2} PZX = \frac{\text{Sin.}\,(s-PZ)\,\text{sin.}\,(s-ZX)}{\text{Sin.}\,PZ\,\text{sin.}\,ZX}$$

where s is the semi-perimeter.

THE PORTABLE TRANSIT INSTRUMENT.

Fig. 12.

This instrument is generally used on extensive surveys, as the local time, latitude, and longitude can be very accurately found by it.

It consists of a telescope mounted on Y's, as shewn in the figure, and adjusted so that its axis moves in a vertical plane, generally that of the meridian. In the eye-piece are one, three, or five vertical wires, illuminated (at night) by a lantern on one of the trunnions—on the other trunnion is a spirit-level and circular arc for setting the telescope at any required angle of elevation. There is also a level for getting the two Y's on the same level.

There are three adjustments for the instrument.

(1) To get the Y's on the same level.

(2) To get the central wire (or the line of collimation) into its correct position as regards the axis of rotation.

(3) To correct for deviation. That is, to make the axis of the telescope move in the plane of the meridian, the axis of rotation lying, of course, exactly east and west when this adjustment has been made.

The first adjustment is made by a spirit level. The third by moving the instrument in azimuth till the central wire intersects a point placed due north or south of it, the meridian line having been previously ascertained. To adjust for collimation the telescope is reversed in its Y's and again turned on the meridian mark. If the central wire now does not coincide with it half the error must be corrected by the collimation screws and half by moving the instrument in azimuth, and the operation repeated till the collimation is perfect.

In taking a transit the time of the object passing each wire is noted, and the mean of all the times taken.

The transit of a star gives the sidereal time of the instant, this being the same as the star's Right Ascension, and the sidereal time being known the mean time can be inferred in the usual way. The longitude can be found very accurately by means of moon-culminating stars, and the latitude by the method given on next page. For all these observations it is necessary to have a good watch, the rate of which is known.

TO FIND THE LATITUDE BY OBSERVATIONS OF STARS ON THE PRIME VERTICAL.

The prime vertical is the great circle passing through the zenith at right angles to the meridian. By means of a transit instrument placed in this position (that is with the vertical arc lying due east and west) the latitude can be very accurately found.

Fig. 13.

Let AB be the meridian, $C'D$ the prime vertical, Z the zenith, P the north pole, and S or S' the star in the prime vertical. Let the sidereal time of the transit be t, and a the star's R. A. Then the hour angle SPZ will be $a-t$. Let d be the star's declination, and l the latitude.

Now in the right-angled triangle PZS we have

$$\text{Cos. } P = \text{tan. } PZ \text{ cotan. } PS$$

$$\text{or Cos. } (a-t) = \text{cotan. } l \text{ tan. } d.$$

This equation gives l very accurately, even with a portable instrument.

If both the east and west transits of a star are taken half the sum of the times ought to be the same as the star's R. A., and the error of deviation may be thus corrected. It should also be noted that if we reverse the telescope on its supports any error of collimation or inequality of pivots will produce exactly contrary effects on the determination of the latitude. The star may be taken with the telescope in reversed positions, either twice the same day, or once on two successive days; the mean of the two latitudes obtained being taken.

If the watch used goes tolerably well the latitude can be found by this method without any reference to sidereal time by taking the time of the east and west transits and reducing the interval from mean to sidereal time. This will give the angle SPS', half of which is SPZ. A star should be chosen which culminates a little south of the zenith, and the telescope should be reversed in its supports between the two transits.

With the instrument called the Zenith Sector the latitude can be found with extreme nicety. This instrument moves in the plane of the meridian and measures the zenith distances of stars which culminate near the zenith (and which are therefore unaffected by refraction) with great accuracy. Certain stars whose declinations are very exactly known are preferred for this. From the measured zenith distance of the star and its known declination we find the zenith distance of the pole, which is equal to the co-latitude.

Fig. 14.

Latitude $=90° - PZ$
$=90° - (\text{star's declination} \pm ZS)$

INTERPOLATION.

In taking out variable quantities from the Nautical Almanac it is necessary to interpolate for the local time and longitude of the place of observation, since the data given are for Greenwich time.

If the rate of change of the variable quantities is itself variable we must allow for this if we wish to obtain a very accurate result.

As an instance: we wish to find the sun's declination at apparent noon on the 2d January, 1880, at a place in longitude 60° or 4h. west.

For Greenwich mean noon we find in the N.A.

Date.	*Sun's declination.*	*Variation in 1 hour.*
2nd January....	22° 57′ 16″·2	13″·21
3rd "	22° 51′ 45″·4	14″·35

Now, at apparent noon at the place it will be 4 P.M. apparent time at Greenwich, and we take the variation at 2 P.M. Greenwich as the *average* variation for those 4 hours.

This variation is 13″·305, which, multiplied by 4 gives 53″·22 to be subtracted from the declination of 2d January—

22° 57′ 16″·2
53″·22
22 56 22 ·98 = required declination.

14.35
13·21
12) 1·14
·095
13·21
13·305 = variation at 2 P.M.
4
53·22

INTERPOLATION BY SECOND DIFFERENCES.

The difference between two consecutive tabulated numbers is called a "first difference," and between two consecutive first differences a "second difference." In many astronomical problems second differences have to be taken into account in interpolating. The method can be best shown by an example or two. The formula employed is

$$f(a+k)=f(a)+Ak+Bk^2.$$

Where A is half the sum of two consecutive 1st differences, and B is half their difference. The signs of A and B must be noted. If the numbers are decreasing the 1st differences are negative, and if the 1st differences are decreasing the 2nd differences are negative.

Ex. 1.—Given the logs. of 365, 366 and 367 to 7 places of decimals to determine log. 366·4.

Numbers.	Log.	1st Differ'ce.	2d Differ'ce.
365 366 367	5622929 5634811 5646661	11882 11850	—32

Here k is $\frac{4}{10}$, A=11866, and B=—16.

```
5634811            11866 × 4/10
   4746                4
-------            -----
3639557            47464
      3
-------            -32/2 × (4/10)² = —3, nearly.
3639554
```

The tables give the log. as 3639555.

If the second difference had been neglected—*i.e.*, if we had worked by simple interpolation, the result would have been 5639551.

Ex. 2.—Given log. cos. of 89° 32′, 89° 33′, and 89° 34′, to find log. cos. 89° 33′ 15″.

	1st Difference	2nd Difference
Log. cos. 89 32=7·9108793 Log. cos. 89 33=7·8950854 Log. cos. 89 34=7·8786953	—157939 —163901	—5962

Here we have to *subtract* $\frac{15}{60}\times$ half the sum of the 1st differences, and $(\frac{15}{60})^2\times$half the second difference, or 40416 in all

∴ log. cos. 89° 33′ 15″=7·8910438.

TO FIND THE LONGITUDE BY TRANSITS OF MOON-CULMINATING STARS.

A surveyor provided with a portable transit telescope and a watch that keeps good time can obtain his longitude wit' considerable accuracy by taking transits of the bright limb of the moon and of certain stars, the R.A. and declination of which are nearly the same as that of the moon at the time. It is not necessary to know either Greenwich or local time. All that has to be done is to set up the telescope in the plane of the meridian, note the times of the transits, and reduce the interval between them from mean to sidereal time.

Fig. 15.

In the N. A. are given, for every day in the year, the sidereal times of transit at Greenwich of the moon and of certain suitable stars, called "moon-culminating" stars; also the rate of change per hour (at the time of transit) of the moon's R.A. As the moon moves rapidly through the stars from west to east it is evident that at a station not on the meridian of Greenwich the interval between the two transits will be different from that at Greenwich; and the moon's rate of motion per hour being known a simple proportion will (if the station is near the meridian of Greenwich) give the difference of time between the station and Greenwich, and thence the longitude. If the station is far from the meridian of Greenwich a correction will have to be made for the change in the rate of change of the moon's R.A. The rate of change at the time of transit at the station is found from the N. A. by interpolation by 2nd differences, and the means of the rates of change at Greenwich and at the station is taken as the rate for the whole interval of time between the transits.

An example will best illustrate the method:

At a certain station to the west of Greenwich on the 25th Oct. the interval between the transits of α Tauri and of the moon's bright limb, reduced to sidereal time, was found to be 0h. 4m. 2s.

First, to find the approximate longitude—

Greenwich Transits.			
α Tauri........	4h.	28m.	18s.
Moon's 2d limb.	4	2	26
		25	52
		4	2
		21	50
		60	
		1310s.	

From the N.A. we find that the moon's R.A. changes 161s.17 per hour.

$\frac{1310}{161 \cdot 17} = 8 \cdot 127$. The approximate longitude is 8h. 7m. 37s.

To find the correct longitude we have to determine the variation of the moon's R.A. in one hour at the station. From the N.A. we find

On the 25th	at lower transit	it was	159s·67	1·50 difference.
" "	upper	" "	161 ·17	0·71 "
" 26th	lower	" "	161 ·88	

By interpolation by 2d differences we find the rate of change at the station to be about 161s·73.

$$\frac{161{\cdot}17+161{\cdot}73}{2}=161{\cdot}45.$$

$$\frac{1310}{161{\cdot}45}=8{\cdot}114$$

And the corrected longitude is 8h. 6m. 50s., which is 47 seconds less than the approximate longitude first found, or a difference of about 9½ miles in the latitude of Kingston.

TO FIND THE LONGITUDE BY LUNAR DISTANCES.

The moon moves amongst the stars from west to east at the rate of about 12° a day. Its angular distance from the sun or certain stars may therefore be taken as an indication of Greenwich mean time at any instant—the moon being in fact made use of as a clock in the sky to show Greenwich mean time at the instant of observation. The local apparent time being also supposed to be known, and thence the local mean time, we have the requisite data for determining the longitude of a station.

In the Nautical Almanac are given for every 3d hour of G.M.T. the angular distances of the apparent *centre* of the moon from the sun, the larger planets, and certain stars, as they would appear from the centre of the earth. When a lunar distance has been observed it has to be reduced to the centre of the earth by clearing it of the effects of parallax and refraction, and the numbers in the N.A. give the exact G. M. T. at which the objects would have the same distance.

It is to be noted that though the combined effect of parallax and refraction increases the apparent altitude of the sun or star, in the case of the moon, owing to its nearness to the earth, the parallax is greater than the refraction, and the altitude is lessened.

Three observations are required—one of the lunar distance, one of the moon's altitude, and one of the other object's altitude. The clock time of the observations must also be noted. The sextant is the instrument used. All the observations can be taken by one observer, but it is better to have three or four. If one of the objects is at a proper distance from the meridian the local mean time can be inferred from its altitude. If it is too near the meridian the watch error must be found by an altitude taken either before or after the lunar observation.

Four or five sets of observations should be made and written down in their proper order.

	Time by watch.	Alt. of star.	Alt. of moon's lower limb.	Dist. of moon's far limb
1st obs'n	....	....	....	....
2nd "	....	....	....	....
3rd "	....	....	....	....
4th "	....	....	....	....
	4)			Totals.
Mean	....	....	....	....

If there is only one observer it is best to take the observations in the following order, noting the time by a watch. 1st, alt. of sun, star or planet; 2d, alt. of moon; 3d, any odd number of distances; 4th, alt. of moon; 5th, alt. of sun, star, or planet. Take the mean of the distances and of their times. Then reduce the altitudes to the mean of the times; *ie.*, form the proportion—difference of times of altitudes : diff. of alts. :: diff. between time of 1st alt. and mean of the times : a fourth number which is to be added to or subtracted from 1st alt. according as it is increasing or diminishing. This will give the altitudes reduced to the mean of the times, or corresponding to that mean.

The altitudes of moon and star must be corrected as usual, and the augmented semi-diameter of the moon added to the distance to give the distance of its centre. The lunar distance has then to be cleared of the effects of parallax and refraction.

TO DETERMINE THE LUNAR DISTANCE CLEARED OF PARALLAX AND REFRACTION.

Fig. 16.

Let Z be the observer's zenith, Zm and Zs the vertical circles in which the moon and star are situated at the instant of observation. Let m and s be their observed places, M and S their places after correction for parallax and refraction: then Zm, Zs and ms are known by observation: ZM and ZS are obtained by correcting the observations. The object of the calculation is to determine MS.

Now, as the angle Z is common to the triangles mZs and MZS, we can find Z from the triangle mZs in which all the sides are known. Next, in triangle MZS there are known MZ, ZS, and the included angle Z, from which MS can be found. MS is the cleared Lunar Distance. The numerical work of this process is tedious.

The cleared distance having been obtained we proceed in accordance with the rules given in the N. A.

The G. M. T. corresponding to the cleared distance can be found either by a simple proportion or by proportional logs.

It admits of proof that if D be the moon's semi-diameter as seen from the centre of the earth (given in N. A.) D' its semi-diameter as seen from a spectator in whose zenith it is, D'' its semi-diameter as seen at a point where its altitude is A, then

$$D''-D=(D'-D) \text{ sin. } A, \text{ very nearly.}$$

Supposing a traveller to have entirely lost his reckoning, but to be furnished with the requisite instruments, viz : Sextant, Artificial Horizon, two Chronometers—one to show Greenwich, the other local time, Nautical Almanac, and set of Nautical Tables. He can proceed as follows :—

First, by means of a meridian altitude of the sun he could find his latitude approximately (not exactly, for not knowing Greenwich mean time he cannot be certain of the sun's declination.)

In the afternoon, by means of a measured altitude, he could determine the error of one of the chronometers which indicates local time. This also would be only approximate, since he uses an approximate value of the latitude, and is ignorant of Greenwich mean time.

If for any reason he cannot take the meridian altitude of the sun he can ascertain the error of the watch by means of equal altitudes. This again would only be an approximate value, since he does not know the rate of his watch. He might also use any two altitudes for the watch error and latitude.

By means of a Lunar Distance he can then determine Greenwich mean time.

Lastly, by repeating the observations, he can determine all the above quantities correctly.